Consistent Comparisons of Real Incomes across Time and Space

by

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Time Space Comparisons of Real Incomes

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Abstract

Consistent real income comparisons over time and space are critical for studies on catch-up and convergence. The paper provides an analytical framework for making real income comparisons across countries and over time which satisfy transitivity and at the same time reflect an underlying nonhomothetic utility function for a representative consumer. The concept of reference price comparisons is developed and implemented using nonhomothetic translog and Almost Ideal Demand Systems. The paper discusses a *direct approach* which uses all the parameters of the demand system to make real income comparisons and an *indirect approach* which adjusts the national price-based comparisons using reduced information on only income elasticities of demand. The proposed approach is empirically implemented using data from the 1980 and 1996 benchmark data from the International Comparison Program and the empirical results report confirm the analytical results discussed in the paper.

Key words: Real income comparisons; constant price comparisons; national price-based comparisons; transitivity; non-homothetic preferences; translog expenditure function; Almost Ideal Demand System;

1. Introduction

Studies on catch-up and convergence of real incomes and comparative assessment of growth performance across countries require consistent estimates of real gross domestic product (GDP). A major source of data on real GDP is the Penn World Tables (PWT) which regularly publishes estimates of real GDP for a large panel of countries covering a fifty-year period. Similar series are also available from the work of Angus Maddison (2007) and the series regularly published the World Development Indicators (World Bank, 2006). In a benchmark year, calculations of real GDP typically depend on cross-country data from the International Comparison Program (ICP). Then to extend the benchmark calculation over time, usually referred to as *real GDP at constant prices*, the conventional practice is to use *observed growth rates in real GDP in respective countries*, taken from national accounts.[1] Therefore, the existing approach combines the use of *real GDP computed using cross-country prices at a point in time* (from a selected ICP benchmark) with *national GDP growth rates* (from national accounts) which typically use country-specific price data. This conventional approach has an obvious inconsistency by mixing cross-country price data in a reference or benchmark year at a point in time with growth rates at national prices over time.[2]

The main objective of this paper is to develop an analytical framework for making real income comparisons *across time and space*. There are several criteria that such a comparison should fulfill. First, the comparison should be transitive across countries and over time. To achieve transitivity, the methodology often used in international comparisons for a single benchmark year is the so-called GEKS (Gini, 1931; Eltetö and Köves; 1964; and Szulc, 1964) method, which extends bilateral comparisons between countries to multilateral comparisons.[3] A specific example is Caves, Christensen and Diewert (CCD, 1982), which makes bilateral comparisons across countries using Törnqvist indexes, and then extends that to a transitive, multilateral comparison by taking averages across countries, as we discuss in the next section.

Second, the real income comparisons should reflect an underlying nonhomothetic utility function for a representative consumer. That criterion is not met by fixed-weight calculations such as Geary-Khamis method, for example, used in the construction of the PWT.[4] But this goal is met by the recent work of Neary (2004), who develops a framework to construct the Geary-Allen International Accounts (GAIA) system. Like Neary, we assume that the cross-country data matches an expenditure function of a representative consumer. In practice the expenditure function may or may not be estimated, and so our goal is to derive some results that make full use of the expenditure function, and other results that do not depend on full knowledge of its parameters, but rely on only the income elasticities of demand. The latter results might be useful, for example, when researchers are trying to reconcile cross-country real income estimates at two points in time, using extraneous information on the income elasticities.[5]

In section 2, we outline the basic framework for real income comparisons with a particular focus on the approaches used in national price and real income comparisons over space and time. The concept of "constant reference price comparisons" is also developed. In section 3 we introduce the expenditure functions for the nonhomothetic translog and Almost Ideal Demand System (AIDS), and use these to obtain "direct" constant price comparisons under full knowledge of the expenditure function. In section 4, we describe national price comparisons along with "indirect" constant price comparisons that use less information on the expenditure function. Section 5 provides the empirical application of the methods proposed in the paper using ICP data for benchmark years 1980 and 1996, and section 6 concludes.

2. Basic Framework

Suppose that there are M countries indexed by $j = 1,2,\ldots,M$; along with N commodities labeled $i = 1,2,\ldots,N$; and two time periods $t-1$ and t. Price and quantity data are represented by vectors $\mathbf{p}_{js} > \mathbf{0}$ and $\mathbf{q}_{js} > \mathbf{0}$ for countries $j=1,2,\ldots,M$ and periods $s = t-1$ and t, and total expenditure in domestic prices is denoted by $z_j = \mathbf{p}_j'\mathbf{q}_j$. Following the economic approach, we make use of an expenditure function depending on prices, $\mathbf{p}$, and a specific utility level u:

$$e(\mathbf{p},u) = Min_{\mathbf{q}}\{\mathbf{p}'\mathbf{q} : U(\mathbf{q}) \geq u\}. \tag{1}$$

All the observed expenditures are assumed to be optimal for the utility levels observed. That is, $z_j = \mathbf{p}_j'\mathbf{q}_j = e[\mathbf{p}_j, U_j(\mathbf{q}_j)]$ in both periods $t-1$ and t.

National-Price based Real Income Comparisons (NPC) across Space and over Time

We begin with a description of the methods currently used in making real income comparisons over time and space. In a given period t, a comparison of real incomes in countries j and k, termed a *national-price based real income comparison,* and abbreviated as *NPC*, is defined as:

$$NPC_{kt}^{jt} \equiv \left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{kt}, u_{kt})}\right] / PPP_{kt}^{jt} \quad , \tag{2}$$

where PPP_{kt}^{jt} which denotes the purchasing-power-parity exchange rate that is used to convert country j national prices into the same units as country k prices. As we discuss in section 4, PPP_{kt}^{jt} should be measured by a price index of country j relative to county $k,$ and in practice is often measured by the Fisher Ideal or Törnqvist prices indexes. The use of such bilateral indexes means that the comparison in (2) is not transitive across countries. We further argue in section 4 that even with the theoretically correct index, (2) as it stands will not give consistent, transitive comparisons in the case of nonhomothetic preferences.

Transitive comparisons can be achieved by using the GEKS method, which transforms bilateral comparisons in (2) through a simple transformation.[6] These are given by:

$$\overline{PPP}_{kt}^{jt} \equiv \prod_{\ell=1}^{M} [PPP_{kt}^{\ell t} \cdot PPP_{\ell t}^{jt}]^{1/M} \ , \tag{3a}$$

$$\begin{aligned} \overline{NPC}_{kt}^{jt} &\equiv \prod_{\ell=1}^{M} [NPC_{kt}^{\ell t} \cdot NPC_{\ell t}^{jt}]^{1/M} \\ &= \left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{kt}, u_{kt})} \right] / \overline{PPP}_{kt}^{jt} \ . \end{aligned} \tag{3b}$$

It is easy to check that the transformed measures of *PPP*s and real income comparisons satisfy transitivity. In that case, we can choose any numeraire country *k* and obtain the real incomes as in (3b). If the bilateral price index in (2) is a Törnqvist index, then the comparison in (3b) is very close to what is recommended by CCD to obtain multilateral comparisons.[7]

Given a comparison like (3b) in a benchmark year, we are interested in extending this over time. As noted in the introduction, a common method is to use the growth rate of real income from the national accounts to extrapolate from a benchmark data. We refer to this as a *national-price based growth comparison*, also denoted by *NPC*, between periods *t-1* and *t* in country *j*, given by:

$$NPC_{jt-1}^{jt} \equiv \left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{jt-1}, u_{jt-1})} \right] / CPI_{jt-1}^{jt}, \tag{4}$$

where CPI_{jt-1}^{jt} denotes a cost-of-living price index for country *j* between periods *t-1* and *t*, which are in practice computed using a Fisher Ideal or Törnqvist price indexes. Again, even with the theoretically correct index, (4) will not give consistent, transitive comparisons in the case of nonhomothetic preferences, as discussed in section 4.

There are two problems with combining spatial comparisons of real incomes in (2) with temporal comparisons in (4). First, the data sources used to make spatial and temporal national-price comparisons are different: the data for *NPC* in (4) comes from national statistical agencies, while the data for *NPC* in (2) comes from international agencies such as the World Bank's International Comparison Program. There may be wide differences in the sampling methodology across these organizations. Even putting aside such differences, there is a second more fundamental problem: when the *NPC* in (2) is done in one benchmark year, and then extrapolated forward using the *NPC* in (4), there is no reason to expect that we will obtain a result that matches the *NPC* done in that future year.[8] The differences between benchmark calculations of real income and the national growth rates are vividly illustrated by Rao, Rambaldi and Doran (2008). Our goal is to achieve such a consistency, by using "constant reference price comparisons."

Constant Price Comparisons (CPC)

In order to achieve consistent comparisons of real income across countries and over time, we propose comparisons based on a reference price vector $\boldsymbol{\pi}$. Under this approach, a *constant-price comparison (CPC)* of incomes across countries for a given time period is:

$$CPC_{kt}^{jt}(\boldsymbol{\pi}) \equiv \frac{e(\boldsymbol{\pi}, u_{jt})}{e(\boldsymbol{\pi}, u_{kt})} \quad . \tag{5a}$$

The constant-price growth comparison for a given country j is:

$$CPC_{jt-1}^{jt}(\boldsymbol{\pi}) = \frac{e(\boldsymbol{\pi}, u_{jt})}{e(\boldsymbol{\pi}, u_{jt-1})} \, . \tag{5b}$$

We may note here that the right hand side of equations (5a) and (5b) are essentially the Allen (1949) quantity indices defined using a reference price vector $\boldsymbol{\pi}$. Obviously, the actual computation of the indices in (5a) and (5b) depends upon the choice of $\boldsymbol{\pi}$ and knowledge of the expenditure function.

The constant price comparisons over space, or across countries, in (5a) are transitive by construction and the temporal comparisons in (5b) satisfy the time reversal test in a two-period case. *If a common vector of reference prices, $\boldsymbol{\pi}$, is used in implementing (5a) and (5b), then the resulting comparisons would be transitive and consistent over time and space,* thereby achieving our first criterion for a consistent time-space comparison. In addition the approach used here can be applied to the case where preferences are nonhomothetic, which is our second criterion.

3. Expenditure Functions and Constant Price Comparisons of Real Income

In this paper, we shall make use of two specifications for the expenditure function. The first is the translog expenditure function:

$$\ln e(\mathbf{p},u) = \ln g(\mathbf{p}) + (\boldsymbol{\beta}'\ln \mathbf{p})\ln u + \eta \ln u + \phi(\ln u)^2 , \qquad (6a)$$

where,

$$\ln g(\mathbf{p}) = \alpha_0 + \alpha' \ln \mathbf{p} + \tfrac{1}{2} \ln \mathbf{p}' \Gamma \ln \mathbf{p} . \qquad (6b)$$

The translog direct and indirect utility functions were introduced by Christensen, Jorgenson and Lau (1975), and the expenditure function in (6) was proposed by Diewert (1976, p. 122). We will simplify the parameters included in the expenditure function by imposing "money metric scaling" (Samuelson, 1974; Diewert, 1980, p. 596). This cardinalization of utility states that at a set of positive prices $\mathbf{p}^*$, expenditure equals utility. We will choose the prices at the unit vector, $\mathbf{p}^* = \mathbf{1}$, and therefore impose the condition:

$$e(\mathbf{1},u) = u . \qquad (7)$$

Substituting (7) into (6) we obtain,

$$\ln e(\mathbf{1},u) = \alpha_0 + \eta \ln u + \phi(\ln u)^2 = \ln u .$$

In order for this condition to hold for all u we must have,

$$\alpha_0 = 0, \quad \eta = 1, \quad \phi = 0 \,. \tag{8}$$

We refer to the parameter restrictions in (8) as "money metric scaling," and assume that it holds henceforth. These conditions are without loss of generality because they do not restrict the demand equations, which are expressed in share form as:

$$\mathbf{s} = \boldsymbol{\alpha} + \boldsymbol{\Gamma} \ln \mathbf{p} + \boldsymbol{\beta} \ln u \,, \tag{9}$$

and correspond to a nonhomothetic utility function if $\boldsymbol{\beta} \neq 0$. Denoting the elements of $\boldsymbol{\Gamma}$ by γ_{ih}, with $\gamma_{ih} = \gamma_{hi}$, then homogeneity of degree one in prices requires that $\sum_i \alpha_i = 1$ and $\sum_i \beta_i = \sum_i \gamma_{ih} = 0$ for each h. To ensure that expenditure is increasing in utility we also require that $1 + \boldsymbol{\beta}' \ln \mathbf{p} > 0$.

The second expenditure function we shall consider is the Almost Ideal Demand (AIDS) system specified as (Banks et al, 1997):

$$\ln e(\mathbf{p}, u) = \ln g(\mathbf{p}) + b(\mathbf{p}) \ln u \,, \tag{10a}$$

where $g(\mathbf{p})$ is as defined in (6b) and, $\quad b(\mathbf{p}) = \eta \prod_i p_i^{\beta_i}$. (10b)

Again, we impose "money metric scaling", which from (7) leads to the following restrictions on the parameters in (10):

$$\alpha_0 = 0, \quad \eta = 1. \tag{11}$$

To ensure that the expenditure function is homogeneous of degree one in prices we require that $\sum_i \alpha_i = 1$ and $\sum_i \beta_i = \sum_i \lambda_i = \sum_i \gamma_{ih} = 0$ for all h, and to ensure that expenditure in increasing in utility we require that $b(\mathbf{p}) > 0$.

Denoting expenditure at domestic prices of country *j* by $z = \mathbf{p'q}$, the expenditure shares for the AIDS are:

$$\mathbf{s} = \boldsymbol{\alpha} + \boldsymbol{\Gamma} \ln \mathbf{p} + \boldsymbol{\beta} \ln[z / g(\mathbf{p})], \tag{12a}$$

where we have eliminated the utility level in (12a) by making the substitution $\ln[z / g(\mathbf{p})] = [b(\mathbf{p}) \ln u]$, from (10a). Note that the income elasticities of demand are:

$$\frac{\partial(s_i z / p_i) / q_i}{\partial z / z} = \frac{\partial(s_i z) / s_i}{\partial z} = 1 + \left(\frac{\beta_i}{s_i}\right). \tag{12b}$$

Thus, given the budget shares, knowledge of the β_i coefficients gives us the income elasticities and vice-versa. Our theoretical results will hold for both the translog and AIDS expenditure functions, but in the empirical application we shall focus on the AIDS function.

Constant Price Comparisons with Specific Demand Systems

Using these two expenditure functions, we are now in a position to address the question as to how the constant-price comparisons are related to underlying utility levels, i.e., how accurately do they describe the relative utility levels in two countries or over time. This question can be answered for the comparisons over time or across countries, as follows.

Theorem 1

The constant-price-change in real income between country-year *jt* and *ks*, for *j=k* or *s=t*, is related to utility levels by:

for the translog case, $$CPC_{ks}^{jt}(\boldsymbol{\pi}) = \left(\frac{u_{jt}}{u_{ks}}\right)^{(1+\boldsymbol{\beta}'\ln \boldsymbol{\pi})}, \tag{13a}$$

and for the AIDS case, $$CPC_{ks}^{jt}(\boldsymbol{\pi}) = \left(\frac{u_{jt}}{u_{ks}}\right)^{b(\boldsymbol{\pi})}. \tag{13b}$$

Proof: In the case translog case, from the definition of *CPC* and from equations (6a), (6b) and (8), we have,

$$\ln CPC_{ks}^{jt} = \ln e(\boldsymbol{\pi}, u_{jt}) - \ln e(\boldsymbol{\pi}, u_{ks})$$
$$= (1 + \boldsymbol{\beta}'\ln\boldsymbol{\pi})(\ln u_{jt} - \ln u_{ks})$$

which establishes (13a). In the AIDS model, from equations (10a) and (11) we have,

$$\ln CPC_{ks}^{jt} = \ln e(\boldsymbol{\pi}, u_{jt}) - \ln e(\boldsymbol{\pi}, u_{ks})$$
$$= b(\boldsymbol{\pi})(\ln u_{jt} - \ln u_{ks})$$

which establishes (13b). Q.E.D.

Remark: In both cases, there is a monotonic transformation of utility, $h(u)$, with $h_u > 0$, such that $CPC_{jt-1}^{jt}(\boldsymbol{\pi}) > CPC_{kt-1}^{kt}(\boldsymbol{\pi})$ if and only if $h(u_{jt})/h(u_{jt-1}) > h(u_{kt})/h(u_{kt-1})$, and $CPC_{kt}^{jt}(\boldsymbol{\pi}) > CPC_{kt-1}^{jt-1}(\boldsymbol{\pi})$ if and only if $h(u_{jt})/h(u_{kt}) > h(u_{jt-1})/h(u_{kt-1})$, so that the constant-price comparison is an accurate measure of the change in utilities. In the translog case we can use the transformation $h(u) = u^{(1+\boldsymbol{\beta}'\ln\boldsymbol{\pi})}$, where $h'(u) > 0$ under the condition $1 + \boldsymbol{\beta}'\ln\mathbf{p} > 0$. Similarly, in the AIDS case, we can use the transformation $h(u) = u^{b(\boldsymbol{\pi})}$ with $b(\mathbf{p}) > 0$.

This result provides a strong justification for the use of constant-price real income in making comparisons across countries and over time. It is clear that these expressions depend upon the choice of $\boldsymbol{\pi}$**.** We can use Theorem 1 to study the differences arising out of different choices of $\boldsymbol{\pi}$**.** Furthermore, when we apply equations (13a) and (13b) on two different reference price vectors, $\boldsymbol{\pi}$ and $\boldsymbol{\pi}^*$**,** this result can also provide an indication as to how we can modify the cross-country comparison $CPC_{kt}^{jt}(\boldsymbol{\pi})$, using reference price vector **π**, to achieve a comparison $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$, using another references price vector $\boldsymbol{\pi}^*$.

To implement Theorem 1 empirically, we should solve for the utility levels appearing on the right of (13). Focusing on the AIDS case and using the parameter restriction in (8), the indirect utility function is:

$$\ln u = [\ln z - \ln g(\mathbf{p})] / b(\mathbf{p}) .$$

Substituting this into (13b) for country-years *jt* and *ks,* we obtain:

$$CPC_{ks}^{jt}(\boldsymbol{\pi}) = \frac{[z_{jt} / g(\mathbf{p}_{jt})]^{b(\boldsymbol{\pi})/b(\mathbf{p}_{jt})}}{[z_{ks} / g(\mathbf{p}_{ks})]^{b(\boldsymbol{\pi})/b(\mathbf{p}_{ks})}} . \tag{14}$$

This formula provides a *direct* calculation of the *CPC,* making full use of the parameters of the AIDS utility function; a related formula can be obtained for the translog case. Then to change from one set of references prices to another, the following result can be used:

Corollary 1

For the translog model, $CPC_{kt}^{jt}(\boldsymbol{\pi})$ can be computed from $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$ by:

$$CPC_{ks}^{jt}(\boldsymbol{\pi}) = [CPC_{ks}^{jt}(\boldsymbol{\pi}^*)]^{\left(\frac{1+\boldsymbol{\beta}'\ln\boldsymbol{\pi}}{1+\boldsymbol{\beta}'\ln\boldsymbol{\pi}^*}\right)} . \tag{15a}$$

For the AIDS expenditure function, $CPC_{kt}^{jt}(\boldsymbol{\pi})$ can be computed from $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$ by:

$$CPC_{ks}^{jt}(\boldsymbol{\pi}) = [CPC_{ks}^{jt}(\boldsymbol{\pi}^*)]^{[b(\boldsymbol{\pi})/b(\boldsymbol{\pi}^*)]} . \tag{15b}$$

The proof of Corollary 1 follows directly from the application of Theorem 1 at reference price vectors **π** and **π*.**

Corollary 1 allows us to obtain the spatial comparisons $CPC_{kt}^{jt}(\boldsymbol{\pi})$ from another $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$ comparison that makes use of **π***. This has the potential to link constant-price real income comparisons with national-price comparisons by making a particular choice of **π*** which depends on the *observed* prices in countries *j* and *k*. A particular choice of $\boldsymbol{\pi}^*$ allows us to establish a link between the constant price real income comparisons, *CPC*, and real income comparisons that make use of national price data which forms the basis for *NPC*. For example, if we consider the geometric mean of national prices, $\pi_i^* = \sqrt{p_{ij} p_{ik}}$, then $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$ is a comparison that is entirely based on the price vectors of countries *j* and *k,* whereas $CPC_{kt}^{jt}(\boldsymbol{\pi})$ depends on some reference price vector that could be determined independent of prices in countries *j* and *k*. Therefore, Corollary 1 provides a link between real income comparisons that depend upon the national price vectors (**π***) and the real income comparison based on any chosen reference price vector, **π**.

Another feature of Corollary 1 is worth noting. The comparison $CPC_{kt}^{jt}(\boldsymbol{\pi})$ where **π** is an arbitrarily selected vector of reference price is transitive over time and space, whereas $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$ which makes use of **π*** that varies for each pair of countries *j* and *k* (similar to the choice $\pi_i^* = \sqrt{p_{ij} p_{ik}}$) will lead to intransitive comparisons. In this case, equation (14) provides a way transforming a set of intransitive bilateral comparisons, $CPC_{kt}^{jt}(\boldsymbol{\pi}^*)$, into a set of transitive comparisons, $CPC_{kt}^{jt}(\boldsymbol{\pi})$. This idea is pursued in the next section to establish a direct link between *CPC* and *NPC*.

4. Real Income Comparisons using National Prices

In the section we suppose that the researcher starts with national prices across multiple countries, such as available for a large number of countries from ICP at benchmark years,[9] along with national sources for all other years. We consider a problem that is similar to the one considered in the previous section: to construct a constant-price comparison of real income across countries and over time but with the difference that the comparisons are based on national prices from various countries. Our goal is to obtain a formula that shows the connection between the *CPC* comparisons developed in the previous section, and the *NPC* calculations that are more typically made.

Consider either the *national-price based real income comparison* for countries *j* and *k*, NPC_{kt}^{jt}, or the *national-price based growth comparison* for country *j*, NPC_{jt-1}^{jt}. These comparisons were given informal definitions in (2) and (4), which depend on index numbers used to compare prices across countries or over time. Specifically, we now define the *NPC* more formally using national prices $\mathbf{p}_{jt}$ and $\mathbf{p}_{ks}$, for $j=k$ or $s=t$, as:

$$NPC_{ks}^{jt} \equiv \left[\frac{e(\mathbf{p}^*, u_{jt})}{e(\mathbf{p}^*, u_{ks})}\right] \equiv \left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{ks}, u_{ks})}\right] / P(\mathbf{p}_{ks}, \mathbf{p}_{jt}, u^*), \tag{16}$$

where $\ln \mathbf{p}^* \equiv \frac{1}{2}(\ln \mathbf{p}_{jt} + \ln \mathbf{p}_{ks})$. The first equality in (16) is the definition of a national-price comparison (over countries or across time): it is a bilateral comparison of expenditure at constant prices $\mathbf{p}^*$, which are a weighted geometric mean of the national prices. The second equality in (16) is the definition of a real income comparison derived by deflating the ratio of observed expenditures in countries j and k using the theoretical price index $P(\mathbf{p}_{ks}, \mathbf{p}_{jt}, u^*)$. In the case of translog or AIDS expenditure functions, we can show that u^* would be a suitably defined weighted mean of u_{ks} and u_{jt}, so that $P(\mathbf{p}_{ks}, \mathbf{p}_{jt}, u^*)$ is a Könus cost-of-living index across countries or time. This is stated and proved in the form of Lemma 1 below.

Lemma 1

For the translog or AIDS expenditure functions, there exists a reference utility level u^* such that the index $P(\mathbf{p}_{ks}, \mathbf{p}_{jt}, u^*)$ in (16) equals,

$$P(\mathbf{p}_{ks}, \mathbf{p}_{jt}, u^*) = \left[\frac{e(\mathbf{p}_{jt}, u^*)}{e(\mathbf{p}_{ks}, u^*)} \right], \tag{17}$$

where the reference utility u^* lies in-between u_{jt} and u_{ks}.

This result is proved in the Appendix, and shows that the theoretical price index in (17) is in fact a Könus cost-of-living index associated with price vectors $\mathbf{p}_{jt}$ and $\mathbf{p}_{ks}$ respectively, and the reference utility u^*. In some cases this Könus cost-of-living index can be readily measured. In the translog case, for example, Diewert (1976, p. 122) shows that the Könus cost-of-living index is the Törnqvist index:

$$\ln P_T \equiv \tfrac{1}{2}(\mathbf{s}_{ks} + \mathbf{s}_{jt})'(\ln p_{jt} - \ln p_{ks}). \tag{18}$$

Conversely, for the AIDS case the Könus cost-of-living index is derived by Feenstra and Reinsdorf (2000).

When comparing *NPC* in (16) and *CPC* in (5), the representative consumer stays on the same indifference curve in each country, and only the prices vary: either national prices or reference prices. For that reason, the connection between *NPC* and the utility levels of the consumer is nearly the same as what we saw for *CPC* in Theorem 1, but using national prices rather than reference prices. We state these results formally as:

Theorem 2

The national-price change in real income between country-year *jt* and *ks*, for *j=k or s=t,* is related to utility levels by:

for the translog case, $$NPC_{ks}^{jt} = \left(\frac{u_{jt}}{u_{ks}}\right)^{(1+\boldsymbol{\beta}'\ln \mathbf{p}^*)}, \quad (19a)$$

and for the AIDS case, $$NPC_{ks}^{jt} = \left(\frac{u_{jt}}{u_{ks}}\right)^{b(\mathbf{p}^*)}. \quad (19b)$$

where $\ln \mathbf{p}^* \equiv \frac{1}{2}(\ln \mathbf{p}_{jt} + \ln \mathbf{p}_{ks})$. Therefore, *NPC* and *CPC* are related by:

for the translog case, $$NPC_{ks}^{jt} / CPC_{ks}^{jt}(\boldsymbol{\pi}) = \left(\frac{u_{jt}}{u_{ks}}\right)^{\boldsymbol{\beta}'(\ln \mathbf{p}^* - \ln \boldsymbol{\pi})}, \quad (20a)$$

and for the AIDS case, $$NPC_{ks}^{jt} / CPC_{ks}^{jt}(\boldsymbol{\pi}) = \left(\frac{u_{jt}}{u_{ks}}\right)^{b(\mathbf{p}^*) - b(\boldsymbol{\pi})}. \quad (20b)$$

Proof: Equations (19a) and (19b) are similar to equations (13a) and (13b) and, therefore, can be shown using steps outlined in the proof of Theorem 1. Equations (20a) and (20b) are derived by taking ratios of (19a) and (13a) and (19b) and (13b), respectively. Q.E.D

While Theorem 2 looks similar to Theorem 1, but using national prices instead of reference prices, there is an important difference: if demand is nonhomothetic so that $\beta_i \neq 0$, then there *does not* exist a monotonic transformation of utility such that when $NPC_{jt-1}^{jt} > NPC_{kt-1}^{kt}$ then the transformation of utility has risen more in country *j* relative to country *k*.[10] Likewise there does not exist a transformation of utility such that when $NPC_{kt}^{jt} > NPC_{kt}^{\ell t}$ then country *j* is better of than country ℓ. In other words, *NPC* is *not* a consistent method to compare utilities across countries or over time (as we asserted in section 2).

The comparison of *NPC* and *CPC* in (20) allows us to investigate whether there is a consistent bias between them. Note that there is no bias if demand is homothetic, in which case $\beta_i = 0$ and the exponents in (20) are zero: in that case and only that case, *NPC* gives a consistent ordering of utilities across countries and over time. Moving beyond that case, suppose that the reference prices $\boldsymbol{\pi}$ are close to the prices of a wealthy country *k,* say the United States, and that these prices are highest for luxury goods, with $\beta_i > 0$. Then the exponents in (20) are *negative:* $\ln p_i^* < \ln \pi_i$ when $\beta_i > 0$, because π_i reflects the high price of luxury good *i* in the U.S. When country *j* has rising utility, it follows that $NPC_{jt-1}^{jt} < CPC_{jt-1}^{jt}(\boldsymbol{\pi})$. In that case, *NPC* will *understate the growth* in real income of country *j*.

On the other hand, suppose instead that the utility of country *j* is falling, or in the spatial context that the utility of country *j* is less than the utility of country *k*. Falling utility might apply to some low-income countries, such as in Africa, when faced with reference prices close to the U.S.; while having lower utility than the U.S. would apply to many countries. Under the same assumptions on prices of luxury goods, the exponents in (20) are still negative. Then for the intertemporal comparison it follows that $NPC_{jt-1}^{jt} > CPC_{jt-1}^{jt}(\boldsymbol{\pi})$, so that *NPC* will *understate the fall* in real income of country *j*. Likewise, in the spatial context we have $NPC_{kt}^{jt} > CPC_{kt}^{jt}(\boldsymbol{\pi})$, so *NPC* will *understate* the extent to which the real income of country *j* is less than *k*.

Putting together these two cases, we see that for the intertemporal comparison using national accounts data, real income growth is *biased towards zero growth* when the reference prices are close to those of a wealthy country with higher prices for luxury goods. This result is confirmed by the empirical results reported in the next section. Similarly, for the spatial comparison using ICP data, the differences in real income *are understated* when the reference prices are close to those of a wealthy country with higher prices for luxury goods. That result is consistent with the finding of Neary (2004) using a quadratic utility function.

Besides solving for the biases between *NPC* and *CPC,* there are also cases where we can solve for one in terms of the other, as shown by the following extension of Theorems 1 and 2.

Corollary 2

In the case of the translog model, $CPC_{kt}^{jt}(\boldsymbol{\pi})$ can be computed as:

$$CPC_{ks}^{jt}(\boldsymbol{\pi}) = (NPC_{ks}^{jt})^{\left(\frac{1+\boldsymbol{\beta}'\ln\boldsymbol{\pi}}{1+\boldsymbol{\beta}'\ln\mathbf{p}^*}\right)} \quad . \tag{21a}$$

For the AIDS expenditure function, $CPC_{ks}^{jt}(\boldsymbol{\pi})$ can be computed from NPC_{ks}^{jt} by:

$$CPC_{ks}^{jt}(\boldsymbol{\pi}) = (NPC_{ks}^{jt})^{[b(\boldsymbol{\pi})/b(p^*)]} \quad . \tag{21b}$$

Since *NPC* is computed from national accounts or international comparisons data, the extra information that we need to obtain *CPC* are the estimates of β_i, reflecting the income elasticities of demand, along with the reference prices. Corollary 2 allows us to obtain the spatial comparisons $CPC_{kt}^{jt}(\boldsymbol{\pi})$ and $CPC_{kt-1}^{jt-1}(\boldsymbol{\pi})$ in two benchmark years, and also obtain the temporal comparisons $CPC_{jt-1}^{jt}(\boldsymbol{\pi})$, *j*=1,…,,M. Because all these comparisons are made at the constant reference prices $\boldsymbol{\pi}$, and are themselves defined as ratios of expenditure functions in (5), it is immediate that we have once again obtained a transitive time-space comparison of real incomes that are transitive and consistent.[11]

Notice that Corollary 2, in conjunction with Lemma 1, provide an *indirect* method to obtain a *CPC* comparison, by first calculating the *NPC* and then applying the transformation in (20). This approach can be contrasted with the *direct* calculation of the *CPC* in (14) for the AIDS case. Whereas the direct calculation in (14) requires all the parameters of the expenditure function, the indirect approach uses only the β parameters, or essentially the income elasticities. There might be circumstances where the reduced information needed on parameters in the indirect approach is useful. For example, the indirect approach might also be useful for, say, economic historians who have cross-country rankings based on a *NPC* calculation at two difference points in time, and who wish to convert this to a time-space consistent ranking using extraneous information on the income elasticities, but without any other parameter estimates.[12] In the following section we will compare the results of the direct *CPC* approach in (14) with the indirect approach using (21).

5. Empirical Application

Data Construction

To illustrate our results, we consider the 1980 and 1996 benchmark years as the two periods (*t-1* and *t*). The data for 1980 are taken from Neary (2004), who adopts the data from Phase IV of the United Nations International Comparison Project (ICP) (United Nations, 1986). There are 60 countries and 11 categories of products in 1980, as shown in Table 1. For 1996, we again use the ICP data, which include 99 countries and 29 product categories. To make consistent comparisons over time, we exclude the last three categories for 1996 (government, construction, and machinery and equipment), and then aggregate the other 26 categories into the same 11 broader categories which are consistent with the 1980 data used by Neary (2004). We focus on the 48 countries appearing in both the 1980 and 1996 datasets.[13]

(Insert Table 1 here)

Estimation and Reference Prices

We follow Neary (2004) and use a semi-flexible approach (Diewert and Wales, 1988) to estimate the AIDS demand system, adapting his GAUSS code.[14] We pool the data for the 48 countries appearing in both years to estimate the AIDS system. In Table 1 we report the estimated parameter values for the AIDS system. In the first column we report the R^2 values for the share equations for each product, and then the parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\Gamma}$.[15] We make use of all these parameters in the *direct CPC* calculations, using (14), but for the *indirect* calculations in (21) we use only the $\boldsymbol{\beta}$ coefficients.

Prior to estimating the AIDS system, we follow Neary's code in first normalizing the country prices for each product by dividing by the mean of prices in the sample, which in our case is 48 countries over both years.[16] Using these normalized prices, the reference price $\boldsymbol{\pi}$**=1** is therefore interpreted as the *sample mean* (over 48 countries and both years) of the product prices. We record these reference prices in Table 2. The U.S. prices will be a second choice for the reference price vector, $\boldsymbol{\pi}=\mathbf{p}_{\mathbf{US}}$, and these are also shown in Table 2 (normalized as discussed, then averaged over 1980 and 1996 and expressed in Table 2 relative to the Misc. category).[17] As a third choice, we can also use the reference prices from the Geary-Allen International Accounts (GAIA) proposed by Neary (2004), $\boldsymbol{\pi}=\boldsymbol{\pi}_{\mathbf{GAIA}}$, which are readily computed from his GUASS code. These are also recorded in Table 2 (again normalized as discussed, and expressed relative to the Misc. category). The GAIA reference prices are weighted more towards lower-income countries than are the U.S. reference prices. At the bottom of Table 2 we compute $b(\boldsymbol{\pi})$ for each of the reference prices vectors, using the $\boldsymbol{\beta}$ estimates from Table 1.

(Insert Table 2 here)

Real Income Comparisons

Table 3 compares different methods used for the cross-nation real income comparisons. We start with the year 1980, so we can have a comparison with Neary (2004). Column (1) gives the actual expenditure per capita in nominal national prices (where for convenience, we have ranked the countries according to their 1996 expenditures). Column (2) gives the *national-price based real income comparison (NPC),* as in (2). The price index PPP_{kt}^{jt} is in fact a Könus cost-of-living index across countries (as discussed in section 4), but is approximated using a Törnqvist index. To make the cross-nation comparison transitive, we follow the GEKS method as defined by (3b). The transitive NPC is reported in column (2) of Table 3, and is defined by using the United States as the comparison country:

$$\overline{NPC}_{us,t}^{jt} \equiv \prod_{\ell=1}^{M} [NPC_{us,t}^{\ell t} \cdot NPC_{\ell t}^{jt}]^{1/M} = \left[\frac{e(\mathbf{p}_{jt}, u_{jt})}{e(\mathbf{p}_{us,t}, u_{us,t})} \right] / \overline{PPP}_{us,t}^{jt} , \qquad (22)$$

with $\overline{PPP}_{us,t}^{jt}$ equal to the multilateral Törnqvist price index, i.e. the GEKS method applied to the bilateral Törnqvist indexes, as in (3a).[18] As expected, the transitive NPC in column (2) shows that the poorer countries are better off than suggested by the simple comparison of expenditure per capita in column (1), because prices are lower in poorer countries.

(insert Table 3 here)

Column (3) of Table 3 then reports the transitive *indirect CPC* across nations. As stated in Corollary 2 and equation (21b), this is measured by utilizing the bilateral *NPC* as well as the knowledge of the estimates of $\boldsymbol{\beta}$ and reference prices. In column (3) we adopt a vector of ones as the reference price vector, i.e., $\boldsymbol{\pi} = \mathbf{1}$, where the vector $\mathbf{p}^*$ is calculated from the geometric mean of national prices for each country pair, $p_{jk,i}^{*} = \sqrt{p_{j,i} p_{k,i}}$. Through those calculations, we obtain the *bilateral indirect CPC*, and then the *transitive indirect CPC* shown in column (3), after the GEKS transformation with the U.S. as the comparison country:

$$\overline{CPC}_{us,t}^{jt} \equiv \prod_{\ell=1}^{M} [CPC_{us,t}^{\ell t} \cdot CPC_{\ell t}^{jt}]^{1/M} = \prod_{\ell=1}^{M} \left[(NPC_{us,t}^{\ell t})^{\frac{b(\boldsymbol{\pi}_{\ell us})}{b(\mathbf{p}_{\ell us}^{*})}} \cdot (NPC_{\ell t}^{jt})^{\frac{b(\boldsymbol{\pi}_{j\ell})}{b(\mathbf{p}_{j\ell}^{*})}} \right]^{1/M} . \qquad (23)$$

The indirect *CPC* in column (3) lowers the measure of real income for the poorer countries, i.e. it offsets the adjustment made in going from nominal expenditure per capital to *NPC*. In many cases, the indirect *CPC* lies in-between the nominal expenditure and *NPC*.

Next, we report several versions of the *direct CPC*, as in equation (14). The results are presented in column (4) through column (7) of Table 3, distinguished by different choices on the reference prices. Column (4) presents a direct comparison to column (3), since it also using a vector $\boldsymbol{\pi} = \mathbf{1}$ as the reference prices; these two columns are quite close to each other. Column (5) uses the U.S. prices as the reference prices, while column (6) uses the GAIA reference prices proposed by Neary (2004). Notice that using the U.S. reference prices results in *lower* estimates of real income for every country as compared to $\boldsymbol{\pi} = \mathbf{1}$**,** whereas using the GAIA reference prices results in *higher* estimates of real income for every country, relative to the U.S. This is exactly as expected from equation (15b), with $[b(\mathbf{p}_{us})/b(\mathbf{1})] = 1.07 > 1$ and $[b(\mathbf{p}_{GAIA})/b(\mathbf{1})] = 0.91 < 1$ used as exponents to compute columns (5) and (6) from column (4). Thus, the *CPC* ranking of countries is unaffected by the choice of reference prices.

The final column (7) in Table 3 uses the geometric mean of national prices (i.e., $p_{jk}^{*} = \sqrt{p_j p_k}$) as the reference prices for each *pair* of countries, and then applies the GEKS transformation to ensure transitivity. In other words, column (7) does not apply a consistent reference price vector across countries. Despite the fact that this approach does not correspond to any of our theoretical results, it is noteworthy that column (7) is quite close to the results with reference prices $\boldsymbol{\pi} = \mathbf{1}$ in the initial *CPC* calculation shown in column (4).

In the first four columns of Table 4, we replicate the results in columns (1) to (4) of Table 3, but changing the year to 1996. As we found for 1980, the income dispersion implied by *NPC* is less than that shown by nominal expenditure per capita, while the indirect *CPC* measure often lies in-between the nominal expenditure and *NPC*. In addition, the indirect *CPC* and direct *CPC*, both with reference prices of $\boldsymbol{\pi} = \mathbf{1}$, are very close. Rather than report further results for other reference prices, in the remaining columns of Table 4 we turn our attention to the intertemporal comparison from 1980 to 1996.

(insert Table 4 here)

Growth Comparisons

First, column (5) of Table 4 gives *the national-price based growth comparison* over the two years, as defined by equation (4), where a Törnqvist price index is used to calculate CPI_{jt-1}^{jt}. A key motivation for this paper, noted in section 2, is that this intertemporal comparison using national prices may very well differ from that obtained by dividing cross-country comparisons at two points in time. To demonstrate this inconsistency, we divide the cross-country *NPC* in column (2) of Table 4 by that in Table 3, and graph that against the growth *NPC* in column (5) of Table 4. This result is shown in Figure 1. The fact that the points in Figure 1 do not line up along the 45 degree line demonstrates the inconsistency between national-price cross-country and intertemporal comparisons. The greatest outlier is Zimbabwe, where the ratio of the cross-country *NPC* is 1.41, whereas the growth *NPC* is 1.19. The cross-country comparison indicates a rise in real income that is *twice as large* (41%) as the growth comparison (19%). Figure 1 illustrates discrepancies between the two measures that are probably much less than would arise in practice, when differing national and international datasets are used for the intertemporal and cross-country calculations, respectively.

(insert Figure 1 here)

Column (6) in Table 4 gives the *indirect* growth *CPC* for 1996 relative to 1980, from equation (21b) (and made transitive using the GEKS transformation). As before, the reference price vector $\boldsymbol{\pi} = \mathbf{1}$, and vector $\mathbf{p}^*$ is calculated from the geometric mean of national prices of the two years, $p^*_{ji} = \sqrt{p_{ji,t} p_{ji,t-1}}$. In column (7) we report the *direct* measure of the growth *CPC* as suggested in equation (14), which is equivalent to directly calculating the ratio of cross-nation direct *CPC* between 1980 and 1996, i.e., dividing column (4) of Tables 3 by that in Table 2.

(insert Figures 2 and 3 here)

The comparison of indirect and direct *CPC* growth is best understood from Figure 2, where we plot the two series. The fact that the points do not lie along the 45 degree line indicates the discrepancy between these two series. The only obvious outlier is again Zimbabwe, probably because of the poor quality of data for African countries. There is an apparent similarity between Figure 1 (comparing intertemporal measures of *NPC*) with Figure 2 (comparing intertemporal measures of *CPC*). This similarity arises because indirect *CPC* growth is a simple transformation of *NPC* growth, as shown by equation (21b). As discussed in the previous section, when the utility of country *j* is rising, we should expect to see $NPC^{jt}_{jt-1} <$ indirect CPC^{jt}_{jt-1}, so the national-price approach *understates* the growth in real expenditure. That prediction is confirmed for most countries in Table 4. On the other hand, when the utility of the country in question is falling we can find $NPC^{jt}_{jt-1} >$ indirect CPC^{jt}_{jt-1}, as shown for a number of African countries. This systematic bias is shown in Figure 3, where the points lie above the 45 degree line for most countries, but below the line for some poorer countries.

6. Conclusions

In this we provide a formal approach to real income comparisons using a set of reference prices and expenditure functions associated with the utility function of a reference consumer. The resulting set of *constant-price comparisons* are transitive and offer consistent temporal-spatial comparisons. The paper also offers a formalization of the current approach followed within the International Comparison Program (ICP) which makes use of the observed prices in different countries. We call this approach *national-price based* comparisons.

A major contribution of the paper is to provide a formal link between the constant-price approach proposed here with the national-price based approach used in the ICP, when the underlying expenditure functions are from nonhomothetic translog and AIDS models. We describe two approaches to constant price comparisons. The *direct* approach uses all the parameters of the demand system to make real income comparisons, while the *indirect* method adjusts the national price-based comparisons (generally derived using the Fisher Ideal or Törnqvist indices) using reduced information on only income elasticities of demand.

The approach is empirically implemented using data from the 1980 and 1996 benchmark data. We confirm that the direct constant-price comparison is transitive across countries and over time: it provides the most consistent ranking, but also requires full knowledge of the expenditure function. In constant, the national-price comparison is inconsistent when we compare the ranking in two years with the intertemporal national-price growth: the ratio of country's real income in two years can deviate substantially from their national-price growth. Some improvement is obtained when we use the indirect constant-price growth, which adjusts the national-price growth for the nonhomotheticity of demand. However, it is evident that the indirect constant-price calculations still differ significantly from the direct constant-price calculations. So the reduced parameter information using in the indirect calculation comes at the cost of reduced accuracy, as compared to the direct calculation.

In summary, we offer a simple and viable method of deriving a transitive set of temporal spatial comparisons based on the constant price real income comparisons. We feel that this approach has considerable potential that should be explored in future research.

Appendix: Proof of Lemma 1

From equation (16), we have,

$$P(\mathbf{p}_{ks},\mathbf{p}_{jt},u^*)=\left[\frac{e(\mathbf{p}_{jt},u_{jt})}{e(\mathbf{p}_{ks},u_{ks})}\right]\Bigg/\left[\frac{e(\mathbf{p}^*,u_{jt})}{e(\mathbf{p}^*,u_{ks})}\right] \tag{A1}$$

where $\ln\mathbf{p}^*=\frac{1}{2}\left(\ln\mathbf{p}_{jt}+\ln\mathbf{p}_{ks}\right)$. In the translog case, using (6a) and (8) we can write (A1) as:

$$[\ln e(\mathbf{p}_{jt},u_{jt})-\ln e(\mathbf{p}^*,u_{jt})]-[\ln e(\mathbf{p}_{ks},u_{ks})-\ln e(\mathbf{p}^*,u_{ks})]$$

$$=[\ln g(\mathbf{p}_{jt})+(1+\boldsymbol{\beta}'\ln\mathbf{p}_{jt})\ln u_{jt}-\ln g(\mathbf{p}^*)-(1+\boldsymbol{\beta}'\ln\mathbf{p}^*)\ln u_{jt}]$$

$$-\left[\ln g(\mathbf{p}_{ks})+(1+\boldsymbol{\beta}'\ln\mathbf{p}_{ks})\ln u_{ks}-\ln g(\mathbf{p}^*)-(1+\boldsymbol{\beta}'\ln\mathbf{p}^*)\ln u_{ks}\right].$$

Substituting $\ln\mathbf{p}^*=\frac{1}{2}\left(\ln\mathbf{p}_{jt}+\ln\mathbf{p}_{ks}\right)$ and rearranging terms we can show that the expression is:

$$\ln g(\mathbf{p}_{jt})-\ln g(\mathbf{p}_{ks})+(1+\boldsymbol{\beta}'\ln\mathbf{p}_{jt})\tfrac{1}{2}(\ln u_{jt}+\ln u_{ks})-(1+\boldsymbol{\beta}'\ln\mathbf{p}_{ks})\tfrac{1}{2}(\ln u_{jt}+\ln u_{ks})$$

$=\ln e(\mathbf{p}_{jt},u^*)-\ln e(\mathbf{p}_{ks},u^*)$ where $\ln u^*=\frac{1}{2}(\ln u_{jt}+\ln u_{ks})$,

which establishes the result for the translog case.

In the AIDS model, we use (10a) and (11) we can write (A1) as:

$$[\ln e(\mathbf{p}_{jt},u_{jt})-\ln e(\mathbf{p}^*,u_{jt})]-[\ln e(\mathbf{p}_{ks},u_{ks})-\ln e(\mathbf{p}^*,u_{ks})]$$

$$=[\ln g(\mathbf{p}_{jt})+b(\mathbf{p}_{jt})\ln u_{jt}-\ln g(\mathbf{p}^*)-b(\mathbf{p}^*)\ln u_{jt}]-\left[\ln g(\mathbf{p}_{ks})+b(\mathbf{p}_{ks})\ln u_{ks}-\ln g(\mathbf{p}^*)-b(\mathbf{p}^*)\ln u_{ks}\right]$$

$$=\ln g(\mathbf{p}_{jt})-\ln g(\mathbf{p}_{ks})+[b(\mathbf{p}_{jt})-b(\mathbf{p}^*)]\ln u_{jt}-[b(\mathbf{p}_{ks})-b(\mathbf{p}^*)]\ln u_{ks}.$$

We can now solve for u^* such that

$$[b(\mathbf{p}_{jt})-b(\mathbf{p}^*)]\ln u_{jt}-[b(\mathbf{p}_{ks})-b(\mathbf{p}^*)]\ln u_{ks}=b(\mathbf{p}_{jt})u^*-b(\mathbf{p}_{ks})\ln u^*. \tag{A2}$$

If $b(\mathbf{p}_{jt})=b(\mathbf{p}_{ks})$ then the Lemma is trivially true for all u^*, so we can choose any utility u^* that lies in-between u_{jt} and u_{ks}. So suppose $b(\mathbf{p}_{jt})\neq b(\mathbf{p}_{ks})$, and without loss of generality consider the case $b(\mathbf{p}_{jt}) > b(\mathbf{p}_{ks})$. By taking logs and using $\ln \mathbf{p}^* = \frac{1}{2}\left(\ln \mathbf{p}_{jt} + \ln \mathbf{p}_{ks}\right)$, we obtain:

$$\ln b(\mathbf{p}^*) = \tfrac{1}{2}[\ln b(\mathbf{p}_{jt}) + \ln b(\mathbf{p}_{ks})] \Rightarrow b(\mathbf{p}_{jt}) > \ln b(\mathbf{p}^*) > b(\mathbf{p}_{ks}). \tag{A3}$$

Then from (A2) we solve for $\ln u^*$ as:

$$\ln u^* = \left[\frac{b(\mathbf{p}_{jt}) - b(\mathbf{p}^*)}{b(\mathbf{p}_{jt}) - b(\mathbf{p}_{ks})}\right]\ln u_{jt} + \left[\frac{b(\mathbf{p}^*) - b(\mathbf{p}_{ks})}{b(\mathbf{p}_{jt}) - b(\mathbf{p}_{ks})}\right]\ln u_{ks}.$$

In this case, the reference utility ln u^* is a weighted average of ln u_{jt} and ln u_{ks}, where the weights are positive and sum to unity, from (A3). QED

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Table 1: Parameter Estimates for the AIDS System

Product	R^2	α	β	Γ										
Food	0.89	0.197	-0.299	-0.007	0.011	-0.029	0.031	-0.017	0.042	0.021	-0.022	0.002	0.021	-0.053
Beverages	0.03	0.029	-0.016	0.011	-0.008	0.016	-0.005	-0.019	-0.013	-0.008	0.001	0.026	0.017	-0.017
Tobacco	0.19	0.019	0.007	-0.029	0.016	0.001	0.021	-0.002	0.015	-0.005	0.005	-0.014	-0.011	0.003
Clothing & Footwear	0.25	0.069	-0.011	0.031	-0.005	0.021	0.005	0.009	-0.014	0.008	-0.008	-0.015	-0.006	-0.026
Gross Rent and Water Charges	0.53	0.109	0.053	-0.017	-0.019	-0.002	0.009	0.008	0.030	0.007	-0.012	-0.007	-0.009	0.012
Fuel and Power	0.35	0.043	-0.001	0.042	-0.013	0.015	-0.014	0.030	-0.044	0.004	-0.001	-0.001	-0.018	0.000
Household Furnishings	0.07	0.069	0.010	0.021	-0.008	-0.005	0.008	0.007	0.004	-0.013	-0.023	0.006	0.002	0.001
Medical and Health Services	0.76	0.086	0.058	-0.022	0.001	0.005	-0.008	-0.012	-0.001	-0.023	0.030	0.017	0.000	0.015
Transport and Communications	0.40	0.116	0.044	0.002	0.026	-0.014	-0.015	-0.007	-0.001	0.006	0.017	-0.004	-0.020	0.009
Recreation and Education	0.55	0.136	0.075	0.021	0.017	-0.011	-0.006	-0.009	-0.018	0.002	0.000	-0.020	0.025	-0.001
Miscellaneous	0.75	0.128	0.081	-0.053	-0.017	0.003	-0.026	0.012	0.000	0.001	0.015	0.009	-0.001	0.057

Notes:

Parameters are estimated from an AIDS specification, using 48 countries over two benchmark years, 1980 and 1996. The first column lists the eleven product categories, and the second column lists the R^2 values for the share equation for each product (over both years), computed as the squared correlation coefficient between the actual and predicted shares (all predicted shares are positive). Following that are the maximum likelihood estimates of **α, β,** and **Γ.**

Table 2: Reference Prices

	$\pi = 1$ (Sample Mean)	$\pi = p_{US}$ (U.S. Prices)	$\pi = p_{GAIA}$ (GAIA prices, Neary, 2004)
Food	1	0.87	1.03
Beverages	1	0.80	0.45
Tobacco	1	0.96	0.68
Clothing & Footwear	1	0.82	1.07
Gross Rent and Water Charges	1	1.00	0.41
Fuel and Power	1	0.78	0.83
Household Furnishings	1	0.96	1.03
Medical and Health Services	1	1.39	0.97
Transport and Communications	1	0.90	0.64
Recreation and Education	1	1.09	0.67
Miscellaneous	1	1.00	1.00
$b(\boldsymbol{\pi})$	1	1.069	0.906

Notes:

Listed are the reference prices used for the calculations for Table 3. Since all prices have been normalized by the sample mean (over 48 countries and two years), the reference prices $\boldsymbol{\pi}$**=1** represents the sample mean. The second set of reference prices considered are those for the U.S. (with the same normalization by the sample mean, then averaged over the 1980 and 1996 years, and shown here relative to the Misc. price = 1). The third set of reference prices considered are the GAIA prices computed as described in from Neary (2004), while pooling the 48 countries and the 1980 and 1996 years (the GAIA prices use the same normalization by the sample mean, and are shown here relative to the Misc. price = 1). The final row is computed using the $\boldsymbol{\beta}$ estimates from Table 1.

Table 3: National-Price and Constant-Price Comparisons, 1980 (U.S.=1)

	Country	Expenditure	*NPC*	Indirect *CPC*	Direct *CPC*			
				π=1	π=1	$\pi=p_{US}$	$\pi=p_{GAIA}$	$\pi=p^*$
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Luxembourg	0.994	0.836	0.880	0.869	0.861	0.881	0.827
2	Denmark	1.122	0.799	0.837	0.840	0.830	0.854	0.802
3	Japan	0.698	0.599	0.646	0.649	0.630	0.676	0.602
4	Norway	0.904	0.650	0.669	0.676	0.658	0.701	0.656
5	United States	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	Germany	1.128	0.833	0.859	0.859	0.850	0.871	0.833
7	Austria	0.890	0.740	0.778	0.784	0.771	0.802	0.746
8	France	1.043	0.814	0.843	0.849	0.840	0.862	0.820
9	Belgium	1.047	0.808	0.830	0.846	0.836	0.859	0.823
10	Finland	0.749	0.611	0.643	0.658	0.639	0.684	0.624
11	Netherlands	0.977	0.781	0.789	0.804	0.792	0.821	0.796
12	United Kingdom	0.804	0.702	0.728	0.735	0.720	0.757	0.708
13	Italy	0.616	0.686	0.722	0.733	0.718	0.755	0.696
14	Hong Kong	0.436	0.623	0.641	0.640	0.621	0.667	0.622
15	Canada	0.886	0.946	0.999	1.021	1.022	1.019	0.967
16	Israel	0.438	0.520	0.527	0.525	0.502	0.558	0.517
17	Ireland	0.481	0.474	0.485	0.508	0.485	0.541	0.494
18	Spain	0.526	0.587	0.595	0.597	0.576	0.627	0.589
19	Greece	0.385	0.460	0.469	0.467	0.443	0.502	0.458
20	Portugal	0.257	0.384	0.389	0.384	0.360	0.420	0.380
21	Korea	0.135	0.190	0.176	0.166	0.147	0.196	0.181
22	Argentina	0.454	0.327	0.320	0.322	0.298	0.358	0.329
23	Uruguay	0.344	0.403	0.408	0.404	0.380	0.440	0.400
24	Chile	0.235	0.299	0.293	0.292	0.268	0.328	0.297
25	Brazil	0.194	0.315	0.308	0.326	0.302	0.362	0.333
26	Hungary	0.149	0.365	0.376	0.374	0.350	0.410	0.363
27	Poland	0.195	0.334	0.339	0.340	0.316	0.376	0.336
28	Dominica	0.124	0.211	0.199	0.196	0.175	0.228	0.208
29	Venezuela	0.297	0.389	0.403	0.401	0.377	0.437	0.387
30	Peru	0.099	0.212	0.201	0.205	0.184	0.238	0.216
31	Panama	0.149	0.223	0.213	0.213	0.192	0.246	0.223
32	Tunisia	0.115	0.173	0.161	0.165	0.146	0.195	0.177
33	Ecuador	0.120	0.201	0.190	0.184	0.164	0.216	0.195
34	Botswana	0.080	0.101	0.091	0.092	0.078	0.115	0.102
35	Philippines	0.066	0.164	0.147	0.141	0.123	0.169	0.158
36	Bolivia	0.088	0.126	0.116	0.122	0.106	0.149	0.132
37	Nigeria	0.081	0.065	0.054	0.056	0.046	0.073	0.067
38	Indonesia	0.039	0.083	0.073	0.075	0.063	0.096	0.085
39	Cote d`Ivoire	0.101	0.095	0.083	0.083	0.070	0.105	0.094
40	Cameroon	0.076	0.078	0.064	0.058	0.048	0.076	0.072

Table 3: National-Price and Constant-Price Comparisons, 1980 (U.S.=1), Continued

	Country	Expenditure	*NPC*	Indirect *CPC*	Direct *CPC*			
				π=1	π=1	$\pi=p_{US}$	$\pi=p_{GAIA}$	$\pi=p^*$
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
41	Zimbabwe	0.060	0.074	0.066	0.071	0.059	0.091	0.079
42	Senegal	0.056	0.069	0.060	0.060	0.049	0.078	0.069
43	Zambia	0.048	0.045	0.037	0.037	0.030	0.050	0.044
44	Madagascar	0.038	0.055	0.046	0.044	0.036	0.059	0.053
45	Kenya	0.037	0.057	0.047	0.047	0.038	0.063	0.056
46	Mali	0.022	0.034	0.026	0.025	0.019	0.035	0.033
47	Malawi	0.019	0.035	0.031	0.035	0.028	0.048	0.039
48	Tanzania	0.027	0.034	0.022	0.017	0.013	0.025	0.028

Notes:

Countries are ranked according to their 1996 values of nominal dollar expenditure per capita. Column (1) gives the nominal expenditure per capita in 1980. Column (2) gives the transitive *NPC*, based on equation (3b) using the Törnqvist price index. Column (3) shows the indirect *CPC* based on equation (21b), using a vector $\boldsymbol{\pi} = \mathbf{1}$ as reference prices. Column (4) to (7) present direct measures of *CPC* based on equation (14), using different reference prices: in column (4) the vector $\boldsymbol{\pi} = \mathbf{1}$; in column (5) the vector of U.S. prices as shown in Table 2; in column (6) the vector of GAIA reference prices, as also shown in Table 2; and in column (7) we use the geometric mean of country prices as the reference vector for each pair of countries. The *NPC*, indirect *CPC,* and direct *CPC* calculation in column (7) are made transitive by applying the GEKS transformation; all other *CPC* calculations are transitive by definition.

Table 4: National-Price and Constant-Price Comparisons, 1996, and Growth Comparisons, 1996/1980 (U.S.=1)

	Country	*NPC* and *CPC,* 1996				Growth, 1996/1980		
		Expenditure	*NPC*	Indirect *CPC* π=1	Direct *CPC* π=1	*NPC*	Indirect *CPC* π=1	Direct *CPC* π=1
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
1	Luxembourg	1.281	0.991	1.027	0.995	1.184	1.195	1.145
2	Denmark	1.219	0.814	0.877	0.849	0.993	1.009	1.011
3	Japan	1.079	0.647	0.741	0.723	1.026	1.068	1.114
4	Norway	1.072	0.705	0.760	0.743	1.049	1.066	1.100
5	United States	1.000	1.000	1.000	1.000	1.000	1.000	1.000
6	Germany	0.996	0.727	0.753	0.755	0.883	0.894	0.879
7	Austria	0.977	0.730	0.757	0.752	0.949	0.962	0.959
8	France	0.938	0.683	0.718	0.715	0.832	0.845	0.843
9	Belgium	0.887	0.704	0.742	0.736	0.868	0.880	0.870
10	Finland	0.828	0.603	0.640	0.631	0.939	0.955	0.960
11	Netherlands	0.761	0.660	0.685	0.684	0.827	0.840	0.850
12	United Kingdom	0.748	0.715	0.758	0.744	0.978	0.991	1.013
13	Italy	0.739	0.688	0.752	0.737	0.979	0.999	1.006
14	Hong Kong	0.718	0.718	0.738	0.720	1.039	1.046	1.126
15	Canada	0.687	0.776	0.823	0.815	0.814	0.827	0.798
16	Israel	0.661	0.608	0.664	0.653	1.136	1.157	1.245
17	Ireland	0.601	0.533	0.569	0.567	1.030	1.048	1.117
18	Spain	0.521	0.506	0.541	0.540	0.851	0.864	0.905
19	Greece	0.458	0.476	0.519	0.513	1.024	1.046	1.099
20	Portugal	0.420	0.478	0.528	0.527	1.290	1.346	1.375
21	Korea	0.317	0.337	0.374	0.361	1.821	2.032	2.172
22	Argentina	0.287	0.356	0.385	0.372	1.116	1.124	1.155
23	Uruguay	0.212	0.280	0.305	0.304	0.715	0.723	0.752
24	Chile	0.157	0.253	0.278	0.265	0.870	0.887	0.908
25	Brazil	0.152	0.198	0.217	0.217	0.645	0.661	0.666
26	Hungary	0.145	0.309	0.344	0.323	0.792	0.799	0.864
27	Poland	0.130	0.238	0.262	0.246	0.672	0.661	0.724
28	Dominica	0.116	0.170	0.182	0.174	0.825	0.837	0.891
29	Venezuela	0.095	0.161	0.178	0.178	0.428	0.416	0.445
30	Peru	0.086	0.151	0.164	0.152	0.656	0.654	0.740
31	Panama	0.084	0.154	0.166	0.158	0.722	0.725	0.739
32	Tunisia	0.065	0.167	0.179	0.156	0.977	1.014	0.941
33	Ecuador	0.054	0.092	0.101	0.100	0.466	0.463	0.545
34	Botswana	0.047	0.109	0.119	0.114	1.143	1.177	1.251
35	Philippines	0.041	0.120	0.129	0.127	0.790	0.797	0.895
36	Bolivia	0.038	0.095	0.102	0.099	0.770	0.781	0.810
37	Nigeria	0.034	0.023	0.019	0.015	0.350	0.298	0.260
38	Indonesia	0.033	0.095	0.105	0.105	1.196	1.226	1.404

Table 4: National-Price and Constant-Price Comparisons, 1996,

and Growth Comparisons, 1996/1980 (U.S.=1),

Continued

	Country	*NPC* and *CPC, 1996*				Growth, 1996/1980		
		Expenditure	*NPC*	Indirect *CPC*	Direct *CPC*	*NPC*	Indirect *CPC*	Direct *CPC*
				π=1	π=1		π=1	π=1
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
39	Cote d`Ivoire	0.024	0.054	0.053	0.045	0.638	0.627	0.549
40	Cameroon	0.024	0.057	0.056	0.048	0.875	0.898	0.827
41	Zimbabwe	0.023	0.088	0.089	0.076	1.409	1.507	1.070
42	Senegal	0.020	0.050	0.048	0.042	0.719	0.718	0.694
43	Zambia	0.014	0.030	0.026	0.021	0.637	0.622	0.566
44	Madagascar	0.013	0.031	0.031	0.028	0.537	0.521	0.647
45	Kenya	0.012	0.040	0.035	0.027	0.701	0.692	0.567
46	Mali	0.011	0.027	0.027	0.024	0.969	0.999	0.931
47	Malawi	0.010	0.027	0.027	0.024	0.794	0.805	0.695
48	Tanzania	0.008	0.019	0.017	0.015	0.584	0.548	0.843

Note:

Countries are ranked according to their 1996 values of nominal dollar expenditure per capita, as shown in column (1). Column (2) gives the transitive *NPC*, based on equation (3b). Column (3) shows the indirect *CPC* based on equation (21b), using a vector $\boldsymbol{\pi} = \mathbf{1}$ as reference prices. Column (4) shows the direct *CPC* based on equation (14), using the reference prices $\boldsymbol{\pi} = \mathbf{1}$. Columns (5) to (7) present the growth ratios between 1980 and 1996: column (5) gives the *NPC* growth, based on equation (4); column (6) gives the indirect *CPC* growth using the $\boldsymbol{\pi} = \mathbf{1}$ as reference prices, based on equation (21b); and finally column (7) gives the direct *CPC* growth using the $\boldsymbol{\pi} = \mathbf{1}$ as reference prices, based on equation (14). The *NPC* and indirect *CPC* calculations are made transitive by applying the GEKS transformation; the direct *CPC* calculations are transitive by definition.

Figure 1: National Price Growth of Real Income

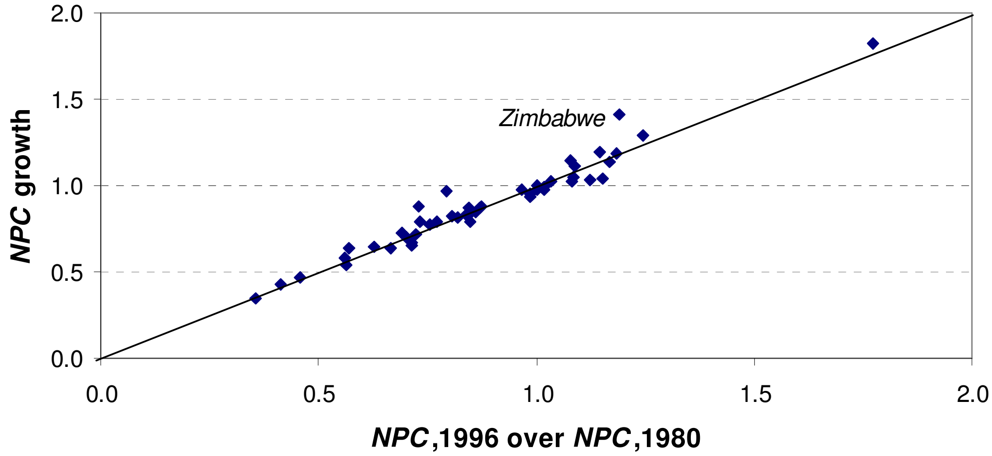


Figure 2: Direct and Indirect Constant Price Growth of Real Income

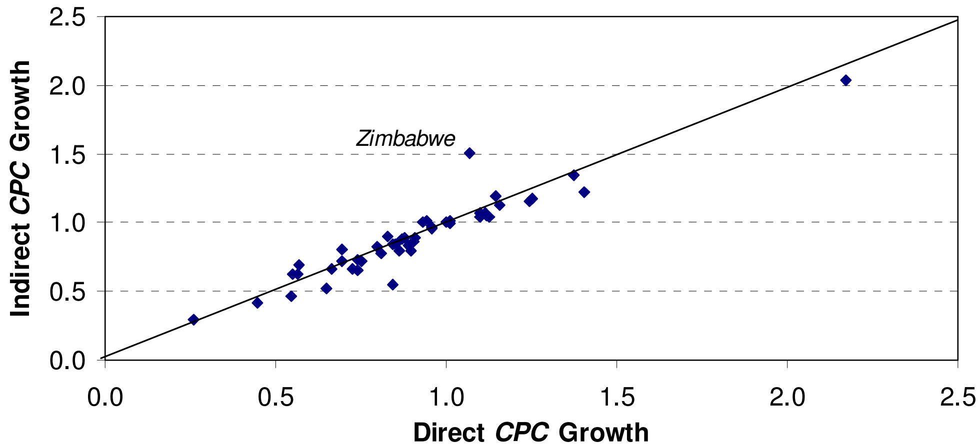
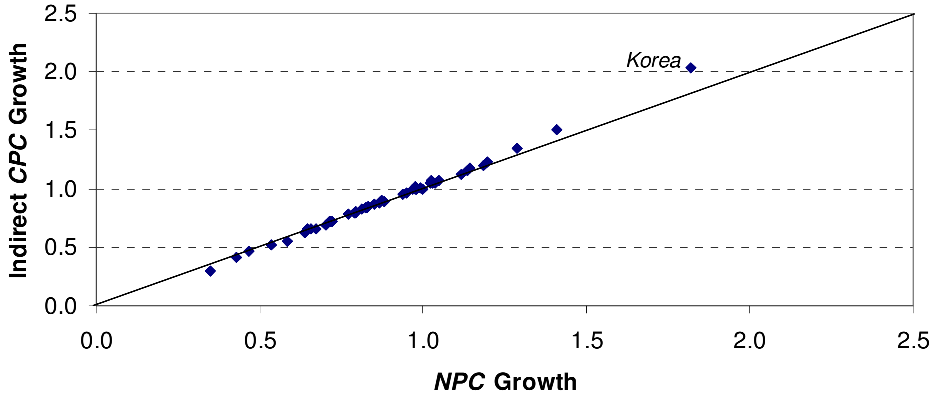


Figure 3: National Price and Indirect Constant Price Growth



[1] More precisely, PWT extrapolates from the benchmark year using growth rates of each component of GDP, i.e. C,I,G and (X-M). In this paper we focus on consumption C.

[2] This inconsistency is also discussed by Feenstra, Heston, Timmer and Deng (2009). A related issue of inconsistency in spatial-temporal comparisons is elaborated in Hill (2004).

[3] A description of the methods is available in the *ICP Handbook* (World Bank, 2005) available on the World Bank website. For further details of the actual implementation the reader is referred to ADB (2007) which provides a comprehensive account in its Report on international comparisons of real GDP in the Asia-Pacific region.

[4] There are methods of computing purchasing power parities using a set of reference prices, like the Geary-Khamis method (Geary, 1958, Khamis, 1970, 1972); Ikle (1972) and Rao (1990). Balk (1996) provides a comparative assessment of these methods. But these methods essentially use fixed-weight calculations and as such have not been favoured in the recent rounds of the ICP.

[5] Oulton (2008) has recently argued that the income elasticities can be estimated with the time-series data typically available to statistical agencies. Our context is different in that we rely on a panel dataset with a cross-section of countries over two years.

[6] This is exactly the approach used in Caves, Christensen and Diewert (1982) in deriving transitive multilateral output comparisons from non-transitive bilateral comparisons based on the Törnqvist index.

[7] CCD use Törnqvist *quantity* indexes to make the comparison in (2) rather than deflating the ratio of expenditures by the Törnqvist *price* indexes, but the difference between these two methods is slight.

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[8] Under the strong assumption of homotheticity such a result would hold (as shown in section 4), but that assumption is usually violated in national consumption data.

[9] The 2005 ICP covered 146 countries from all regions of the world.

[10] To make that statement true, we would need to define the transformation as depending on $\mathbf{p}^*$ and therefore on the national prices of countries j and k, which destroys the comparability of utility across more than two countries.

[11] There is a difference in practice between the transitivity of the "direct" *CPC* in (14), and the "indirect" *CPC* from Corollary 2. In both cases, transitivity requires that the data satisfy expenditure-minimization as in (1). Because the direct calculation in (14) makes full use of the estimated expenditure function, it will automatically satisfy (1), so transitivity is ensured. But the indirect calculation in Corollary 2, making use of the *NPC* in Lemma 1, will not automatically satisfy (1), so in that case we use the GEKS transformation to obtain transitivity in practice.

[12] Oulton (2008) argues that the $\boldsymbol{\beta}$ parameters of the AIDS function can be estimated with the time-series data for a single country typically available from national statistical agencies, without a precise estimate of the substitution parameters $\boldsymbol{\Gamma}$ entering $g(\mathbf{p})$. Our context is different, however, because we rely on cross-country data over two years.

[13] Since prices are initially all measured relative to the U.S. for our data (i.e., the U.S. prices for all 11 categories are equal to unity, for both years), we adjusted the 1996 price matrix by multiplying by the U.S. CPI growth over 1980-1996 for each product. So the 1980 price matrix has the U.S. prices equal to unity, whereas the 1996 price matrix has the U.S. prices above unity reflecting inflation for each product, and then prices for all other countries are still measured relative to those for the U.S.

[14] See Neary (2004, Appendix D), available at:

http://www.economics.ox.ac.uk/members/peter.neary/gaia/gaia.htm .

[15] Neary (2004) estimates the AIDS system using maximum likelihood methods, so the R^2 calculation is made afterwards to give an idea of the fit of the share equations for each product (over both years). The R^2 is computed as the squared correlation coefficient between the actual and predict shares (all predicted shares are positive).

[16] This procedure is applied *after* the adjustments in note 10, and essentially changes the units for each product.

[17] The U.S. prices are measured *after* the adjustments explained in notes 10 and 13, are then averaged over 1980 and 1996, and expressed in Table 1 after normalizing the Miscellaneous price to unity.

[18] Though not reported, we have confirmed that the CCD real income recommended by Caves, Christensen and Diewert (1982), using the multilateral Törnqvist quantity index, is very close to the transitive *NPC* in (22).